

Ch. 17: Probability Models (Geometric and Binomial)

Opening question:

I have a game in which you spin a spinner for your prize. The spinner has 4 sections: Red 45%, Green 30%, Yellow 10% and Blue 15%. Answer the following questions:

- 1- What is the probability of getting ^{1st} red on the 3rd try? $P(R^c \text{ and } R^c \text{ and } R) = (0.55)(0.55)(0.45) = 0.136125$
- 2- What is the probability of getting ^{1st} red on the 7th try? $P(R^c \& R^c \& R^c \& R^c \& R^c \& R^c \& R) = (0.55)(0.55)(0.55)(0.55)(0.55)(0.55)(0.45) = 0.01245$
- 3- What is the probability of getting ^{1st} red on the 20th try? $= (0.55)^{19}(0.45) = 5.249 \times 10^{-6}$
- 4- What is the probability of getting ^{1st} red on the 50th try? $= (0.55)^{49}(0.45) = 8.53 \times 10^{-14}$
- 5- What is the probability of getting ^{1st} blue on the 18th try? $= (0.85)^{17}(0.15) = 0.0095$

How can I simplify my process for any of the above questions?

$$(\text{prob. failure})^{x-1} (\text{prob. success})$$

This type of question is a specific type of probability, and has a specific model:

GEOMETRIC MODELS

* Continue trials until you get a success

- There are 4 conditions that must apply to the trials
 - 2 possible outcomes
 - Probability of success remains constant (called p)
 - Number of trials is UNKNOWN

* Independent trials**

** = 10% Condition: If we cannot assume independence, we can proceed as long as the sample is smaller than 10% of the population

success/failure

$$pop \geq 10 \cdot n$$

So, using our example above, let's derive the formulas associated with GEOMETRIC PROBABILITIES:

- 1- What is the probability of getting a red on the 3rd try?
- 2- What is the probability of getting a red on the 20th try?
- 3- What is the probability of getting a blue on the 18th try?

For GEOMETRIC MODELS.....

$$P(X = x) = (1-p)^{(x-1)} * p$$

or

$$P(X = x) = q^{(x-1)} * p$$

← success

← failure

Examples:

1) I want to roll a die until I get a 6.

a. Is this a Geometric probability? Does it fit our 4 conditions? **YES!!**

6, not 6

unknown n

$$p = 1/6$$

indep.

b. Find the probability that I get a 6 on the 3rd roll

$$P(X=3) = (5/6)^2 (1/6) = 0.1157$$

c. Find the probability that I get a 6 on the 6th roll

$$P(X=6) = (5/6)^5 (1/6) = 0.06698$$

d. Find the probability that I get a 6 on the 10th roll

$$P(X=10) = (5/6)^9 (1/6) = 0.0323$$

e. Find the probability of getting a 6 in the first 4 rolls

$$P(X=1) + P(X=2) + P(X=3) + P(X=4) = 0.5178$$

$1/6 + 0.1389 + \dots$

2) Complete problems #1, 7, 9 from the book (p. 401- 402)

- **VOCAB:** Bernoulli trial = Something that fits the 4 conditions listed above for a geometric model
- **VOCAB:** Geometric models are models in which we continue until we get a success

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GEOMETRIC MODELS:

Expected value: $E(X) = \mu_X = \frac{1}{p}$

Standard deviation: $SD(X) = \sigma_X = \sqrt{\frac{q}{p}} = \sqrt{\frac{(1-p)}{p}}$

prob success

3) Complete #11, 13 in the book (p. 402)

HW: p. 401 #2, 8, 10, 12, 14, 15